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ON πμ*g-CLOSED SETS IN IDEAL GENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

We introduce the notions of πμ* g-closed sets by using the notion of μ-pre-I-open sets. Further, we study the concept of πμ* g-closed sets and their relationships in an ideal generalized topological spaces by using these new notions.

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I. INTRODUCTION

A subfamily μ of the power set *P*(X) of a nonempty set X is called generalized topology *[1]* on X if and only if $\Phi \in \mu$ and $U_i \in \mu$ for $i \in I$ implies $\cup_{i\in I} U_i \in \mu$. We call the pair (X, μ) a generalized topological spaces (briely GTS) on X. The members of μ are called μ-open sets *[1]* and the complement of a μ-open is called a μ-closed set. For $A \subset X$, we denote by μCl(A) the intersection of all μ-closed sets containing A; and by μInt(A) the union of all μ-open sets contained in A. The concept of ideals in topological spaces has been introduced and studied by kuratowski *[4]* and Vaidyanathansamy *[6]*. An ideal I is a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$ *[3]*. With respect to the generalized topology μ of all μ-open sets and an ideal I, for each subset A of X, a subset $A^{*\mu}(I)$ or simply $A^{*\mu}$ of X is denoted by $A^{*\mu} = \{x \in X : U \cap A \in \not\in I$ for every $U \in \mu$ such that $x \in U\}$ *[3]*.

Lemma 1.1 *[5]* Let (X, μ) be a generalized topological space with an ideal I on X and A a subset of X. Then we have the following:

(a) $A^{*\mu}(\mu, \{\Phi\}) = \mu Cl(A)$.
(b) $A^{*\mu}(\mu, P(X)) = \Phi$.
c. If $A \in I$, then $A^{*\mu} = \Phi$.
d. Neither $A \subset A^{*\mu}$ nor $A^{*\mu} \subset A$.

Lemma 1.2 ***[5]*** Let (X, μ) be a generalized topological space with an ideal I on X and A, B a subsets of X. Then we have the following:

a. If $A \subset B$, then $A^{*\mu} \subset B^{*\mu}$
b. $A^{*\mu} = \mu Cl(A^{*\mu}) \subset \mu Cl(A)$ and $A^{*\mu}$ is μ-closed set in (X, μ)
(c) $(A^{*\mu})^{*\mu} \subset A^{*\mu*}$

(d) $(A \cup B)^{*\mu} = A^{*\mu} \cup B^{*\mu}$
(e) $A^{*\mu} - B^{*\mu} = (A - B)^{*\mu} - B^{*\mu} \subset (A - B)^{*\mu}$
(f) If $C \in I$, then $(A - C)^{*\mu} \subset A^{*\mu} = (A \cup C)^{*\mu}$.

Lemma 1.3 *[5]* Let (X, μ) be a generalized topological space with ideals I_1 and I_2 on X and A subset of X. Then we have the following:
(a) If $I_1 \subset I_2$, then $A^{*\mu}(I_2) \subset A^{*\mu}(I_1)$.
(b) $A^{*\mu}(I_1 \cap I_2) = A^{*\mu}(I_1) \cup A^{*\mu}(I_2)$.

Lemma 1.4 *[5]*The set operator μCl* satisfies the following:
(a) $A \subset \mu Cl^*(A)$.
(b) $\mu Cl^*(\emptyset) = \emptyset$ and $\mu Cl^*(X) = X$.
(c) If $A \subset B$, then $\mu Cl^*(A) \subset \mu Cl^*(B)$.
(d) $\mu Cl^*(A) \cup \mu Cl^*(B) \subset \mu Cl^*(A \cup B)$.

Definition 1.5 Let (X, μ) be a generalized topological space with an ideal I on X. A subset
A of X is called
a. μ-α-I-open if $A \subset \mu Int(\mu Cl^*(\mu Int(A)))$.
b. μ-semi-I-open if $A \subset \mu Cl^*(\mu Int(A))$.
c. μ-pre-I-open if $A \subset \mu Int(\mu C^*(A))$.
d. μ-I-regular-open if $A = \mu Int(\mu Cl^*(A))$.

I.$\pi\mu * g$**-closed sets**

Definition 2.1 Let (X, μ) be a generalized topological space with an ideal I on X. A subset H of X is said to be πμ∗ g-closed if $\mu Cl*(\mu Int(H)) \subset U$, whenever $H \subset U$ and U is μ-pre- I open.

Example 2.1 Let X = {a, b, c}, μ = {∅, {a, b}, {a, c}, X} and I = {∅, {a}, {b}, {a, b}}. Then H = {a, c} is πμ∗ g-closed.

Definition 2.2 Let (X, μ) be a generalized topological space with an ideal I on X. A subset H of X is said to be πμ∗ g-open if the complement of H is πμ∗ g-closed in X.
Example 2.2 In Example 2.1, H = {b} is πμ∗ g-open.

Proposition 2.1 Let (X, μ) be a generalized topological space with an ideal I on X. For any $H \in I$,
H is πμ∗ g-closed.

proof: Let $H \subset U$, where U is μ-pre-I open. Since $H* = \emptyset$ for every $H \in I$, then $\mu Cl*(H) = H$. Now $\mu Int(H) \subset H$ implies that $\mu Cl*(\mu Int(H)) \subset \mu Cl*(H) = H \subset U$. Hence for every $H \in I$, H is πμ∗ g-closed.

Proposition 2.2 Let (X, μ) be a generalized topological space with an ideal I on X and $H \subset X$. If H is μ-pre-I open and πμ∗ g-closed, then H is μ-semi-I-closed.

Proof: Let H be μ-pre-I open and πμ∗ g-closed. Let $H \subset H$ where H is μ-pre-I open. Since H is πμ∗ g-closed, $\mu Cl*(\mu Int(H)) \subset H$. Hence H is μ-semi-I-closed.

Proposition 2.3 Let (X, μ) be a generalized topological space with an ideal I on X. Every μ*-closed is πμ∗ g-closed.

Proof: Suppose that H is μ*-closed in X. Let $H \subset U$ where U is μ-pre-I open. Since H is μ*-closed, $\mu Cl*(H) = H \subset U$ and $\mu Cl*\mu Int(H) \subset \mu Cl*(H)$, we get $\mu Cl*(\mu Int(H)) \subset U$, thus H is πμ∗ g-closed.

Proposition 2.4 Let (X, μ) be a generalized topological space with an ideal I on X and H and F be subsets of X. If H and F are πμ∗ g-closed sets, then H ∩ F is πμ∗ g-closed.

Proof: Let H ∩ F ⊂ U where U is μ-pre-I open. Since H and F be πμ∗ g-closed sets in X, we have μCl*(μInt(H))⊂U and μCl*(μInt(F))⊂U. Hence μCl*(μInt(H∩F))⊂μCl*(μInt(H)) ∩ μCl*(μInt(F))⊂ U this implies H ∩ F is πμ∗ g-closed set.

Remark 2.1 The following example shows that the union of two πμ∗ g-closed sets need not be πμ∗ g-closed.

Example 2.3 Let X = {a, b, c, d}, μ = {∅, {a, c}, {a, d}, {a, c, d}} and I = {∅, {c}, {d}, {c, d}}. Then H = {a} and F = {c} are πμ∗ g-closed sets. But H ∪ F = {a, c} is not πμ∗ g-closed.

Proposition 2.5 Let (X, μ) be a generalized topological space with an ideal I on X and H ⊂ X. If H is πμ∗ g-closed, then μCl*(μInt(H))− H contains no nonempty μ-pre-I closed set.

Proof: Suppose that F is a nonempty μ-pre-I closed set of μCl*(μInt(H))-H. Now F⊂ μCl*(μInt(H)) − H implies that F ⊂ μCl*(μInt(H)) ∩ H^c. Hence F ⊂ μCl*(μInt(H)). Now F ⊂ H^c implies that H ⊂ F^c. Since F^c is μ-pre-I open and H is πμ∗ g-closed, we have μCl*(μInt(H)) ⊂ F^c and F ⊂ $(μCl^*(μInt(H)))^c$. Therefore F ⊂ (μCl*(μInt(H))) ∩ $(μCl^*(μInt(H))^c$ = ∅. That is, F = ∅. Thus μCl*(μInt(H)) − H contains no non-empty μ-pre-I closed.

Corollary 2.1 Let (X, μ) be a generalized topological space with an ideal I on X and H be πμ∗ g-closed subset of X. Then H is regular open if and only if μCl*(μInt(H)) − H is μ-pre-I closed.

Proof: Let H be πμ∗ g-closed. If H is regular open, then we have μCl*(μInt(H)) − H = ∅ which is μ-pre-I closed set. Conversely, let μCl*(μInt(H)) − H be μ-pre-I closed. Then, by Theorem 2.5, μCl*(μInt(H)) − H does not contain any nonempty μ-pre-I closed subset of X and since μCl*(μInt(H)) − H is μ-pre-I closed subset of itself, then μCl*(μInt(H)) − H = ∅. This implies that H = μCl*(μInt(H)) and so H is μ-I-regular open.

Proposition 2.6 Let (X, μ) be a generalized topological space with an ideal I on X. Sup- pose that K ⊂ H ⊂ U, K is πμ∗ g-closed relative to H and H is both regular open and πμ∗ g-closed subset of U, then K is πμ∗ g-closed relative to U.

Proof: Let K ⊂ U and U be μ-pre-I open in U. Given K⊂H⊂U. This implies that K ⊂H∩U. Since K is πμ∗ g-closed relative to H, μCl*(μInt(K)) ⊂ H ∩ tt. Therefore, H ∩(μCl*(μInt(K))) ⊂ H ∩U. Consequently, H∩(μCl*μInt(H)) ⊂ U. Since H is regular open and πμ∗ g-closed, we have H = μCl*(H). Therefore μCl*(μInt(K)) ⊂ μCl*(K) ⊂ μCl*(H) = H. Thus μCl*(μInt(K)) ∩ H = μCl*(μInt(K)) and μCl*(μInt(K)) ⊂ U. Hence K is πμ∗ g-closed relative to U .

Corollary 2.2 Let (X, μ) be a generalized topological space with an ideal I on X. Let H be both regular open and πμ∗ g-closed in U and suppose that F is μ-pre-I closed, then H ∩ F is πμ∗ g-closed.

Proof: We have show that μCl*(μInt(H ∩ F)) ⊂ U whenever H ∩ F ⊂ U and tt is μ-pre-I open. Since F is μ-pre-I closed, H ∩ F is μ-pre-I closed in H and hence πμ∗ g-closed in H. Hence H ∩ F is πμ∗ g-closed in U.

Proposition 2.7 Let (X, μ) be a generalized topological space with an ideal I on X. and H ⊂ T ⊂ S. Suppose that H is πμ∗ g-closed in S and T is μ-open, then H is πμ∗ g-closed relative to T.

Proof: Given H ⊂ T ⊂ S and H is πμ∗ g-closed. Let H ⊂ T ∩ U where U is μ-pre- I open. Since H is πμ∗ g-closed, H ⊂ U implies that μCl*(μInt(H)) ⊂ U. Therefore, T ∩ μCl*(μInt(H)) ⊂ T ∩ U. Thus H is πμ∗ g-closed relative to T.

Proposition 2.8 Let (X, μ) be a generalized topological space with an ideal I on X and H ⊂ X. Then H is $\pi_\mu *$ g-open if and only if F ⊂ μInt(μCl * (H)) whenever F is μ-pre-I-closed and F ⊂ H.

Proof: Assume that H is $\pi_\mu *$ g-open, then H^c is $\pi_\mu *$ g-closed. Let F be a μ-pre-I-closed in H contained in H. Then F^c is a μ-pre-I-open set in X containing H^c. Since Hc is $\pi_\mu *$ g-closed, μCl * (μInt(H^c)) ⊂ F^c. Consequently F ⊂ μInt(μCl * (H)).
Conversely, let ⊂ μInt(μCl * (H)) whenever F ⊂ H and F is μ-pre-I-closed in X. Let H be μ-pre-I-open containing H^c, then G^c ⊂ μInt(μCl * (H)). Thus μCl * Int(H^c) ⊂ G. This implies that H is $\pi_\mu *$ g-open.

Remark 2.2 The notions of $\pi_\mu *$ g-closed and μ-semi-I-closed are independent of each other as shown in the following example.

Example 2.4 Let X = {a, b, c}, μ = {∅, {a}, {a, c}, {b, c}, X} and I = {∅, {a}}. Then A = { c} is πμ∗ g-closed but not μ-semi-I-closed.

Example 2.5 Let X = {a, b, c}, μ = {∅, {b}, {a, c}, {b, c}, X} and I = {∅, {a}, {c}, {a, c}}. Then A = { a, b} is μ-semi-I-closed but not πμ∗ g-closed.

Remark 2.3 The notions of πμ∗ g-closed and μ-β-I-closed are independent of each other as shown in the following example.

Example 2.6 Let X = {a, b, c}, μ = {∅, {a}, {a, c}, {b, c}, X} and I = {∅, {b},{c},{a, c}}. Then A = {a, c} is πμ∗ g-closed but not μ-β-I-closed
Example 2.6 Let X = {a, b, c,d}, μ = {∅, {a}, {b, c},{c,d}, {a, b, c}, {a,c,d}, {b,c,d}, X} and I = {∅, {c},{d},{ c, d}}. Then A = {a, b,d} is μ-β-I-closed but not πμ∗ g-closed.

Proposition 2.9 Let (X, μ) be a generalized topological space with an ideal I on X and H ⊂ X. If H is πμ∗ g-closed and H ⊂ K ⊂ μCl*(μInt(H)), then K is also πμ∗ g-closed.

Proof: Let K ⊂ tt where U is μ-pre-I open. Now H ⊂ K implies that H ⊂ U and tt is μ-pre-I open. Since H is πμ∗ g-closed, then μCl*(μInt(H)) ⊂ U. Using hypothesis, μCl*(μInt(K)) ⊂ U. Thus K is πμ∗ g-closed.

Proposition 2.10 Let (X, μ) be a generalized topological space with an ideal I on X and A, B be subsets of X. If μInt(μCl*(A)) ⊂ B ⊂ A and A is πμ∗ g-open, then B is πμ∗ g-open.

Proof: Let μInt(μCl*(A)) ⊂ B ⊂ A. Then X − A ⊂ X − B ⊂ X − μInt(μCl*(A)) = μCl*(μInt(X − A)). Since X − A is πμ∗ g-closed, Proposition 2.9, X − B is πμ∗ g-closed. Hence B is πμ∗ g-open.
Proposition 2.11 Let (X, μ) be a strong generalized topological space with an ideal I on For each a ∈ X, either {a} is μ-pre-I closed or {a}c is πμ∗ g-closed.

Proof: Suppose {a} is not μ-pre-I closed in X. Then {a}c is not μ-pre-I open and the only μ-pre-I open set containing {a}c is X ⊂ X. That is, {a}c ⊂ X. Therefore, μCl*(μInt({a})) ⊂ X, Which implies {a}c is πμ∗ g-closed.

Proposition 2.12 Let (X, μ) be a generalized topological space with an ideal I on X and A ⊂ X. Then A is πμ∗ g-open if and only if F ⊂ μ Int (μCl * (A)) whenever F is μ-pre-I closed and F ⊂ A.

Proof: Suppose that A is πμ∗ g-open. Let F ⊂ A and F is μ-pre-I-closed. Then X − A ⊂ X − F and X − F is μ-pre-I open. Since X − A is πμ∗ g-closed, then μCl*(μInt(X − A)) ⊂ X − F and X − μCl*(μInt(A)) = μCl*(μInt(X − A)) ⊂ X − F and hence F ⊂ μInt (μCl * (A)). Conversely, let X − A ⊂ U where U is μ-pre-I open. Then X − U is μ-pre-I closed. By hypothesis, we have X − U ⊂ μ Int(μCl * (A)) and hence (X − A)* ⊂ μCl* (μInt(X – A)) = X − μInt(μCl * (A)) ⊂ U. Therefore X − A is πμ∗ g-closed and A is πμ∗ g-open.

Proposition 2.13 Let (X, μ) be a generalized topological space with an ideal I on X and A ⊂ X. Then A is πμ∗ g-open iμ(μCl ∗ (A)) ⊂ B ⊂ A, then B is πμ∗ g-open.

Proof: Since A is πμ∗ g-open, then X −A is πμ∗ g-closed. By Proposition 2.4, μCl*(μInt(X − A)) ⊂ X − A contains no nonempty μ-pre-I closed set. Since μInt(μCl ∗ (A)) ⊂ μInt(μCl ∗ (B)), we have X− μInt(μCl ∗ (X − A)) ⊂ X − μInt(μCl ∗ (X − B)), which implies that μInt(μCl ∗ (X – B)) ⊂ μ Int(μCl ∗ (X − A) and so μInt(μCl ∗ (X − B)) − (X − B) ⊂ μInt(μCl ∗ (X – A)) − (X − A). Hence B is πμ∗ g-open.

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